

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – NOVEMBER 2015

ST 1503/ST 1501 - PROBABILITY AND RANDOM VARIABLES

Date : 12/09/2015

Dept. No.

Max. : 100 Marks

Time : 01:00-04:00

Section A

Answer ALL the questions:

(10x2=20)

1. What are equally likely events? Give examples.
2. Write down the three axioms of probability.
3. Mention any two limitations (demerits) of classical probability.
4. Define empirical probability.
5. Explain pair-wise and mutual independence of any three events.
6. For any event 'A' in a sample space S, show that A and S are independent.
7. Explain posterior probability.
8. When do we say that two events are independent?
9. Define random variable. Give one example.
10. Show that the variance of a random variable is always non-negative

Section B

Answer any FIVE questions:

(5x8=40)

11. For any two events A and B, show that
 - (a) $P(A^c \cap B) + P(A \cap B) = P(B)$
 - (b) $P(A \cap B^c) \leq P(A)$
12. A box contains 6 White, 4 Red and 9 black marbles. If 3 marbles are drawn at random, find the probability that (a) Two of the marbles drawn are white. (b) None of the marbles is white
13. In an experiment of throwing two dice at a time, write down the sample space, and find the probability that (a) both dice show the same number, (b) the second dice shows 5, (c) the sum of the numbers on the dice is 11
14. Illustrate through an example that pair-wise independence does not imply mutual independence.

15. State and prove the multiplication law of probability for 'n' events.
16. Write down the sample space associated with the experiment of tossing a coin twice. Obtain the probability mass function of the number of heads that turns up. Find the mean and variance.
17. Sixty per cent of the employees of the XYZ Corporation are college graduates. Of these, ten per cent are in sales. Of the employees who did not graduate from the college, eighty per cent are in sales.
- (a) What is the probability that an employee selected at random is in sales? (b) What is the probability that an employee selected at random is neither in sales nor a college graduate?

18. If X has probability mass function as follows

X	1	2	3	4	5	6	7
P(x)	$\frac{k}{6}$	$\frac{k}{2}$	$\frac{k^2}{3}$	$\frac{k^2 + k}{6}$	$\frac{k^2}{6}$	$\frac{k}{2}$	$\frac{k}{3}$

Find k, E(X) and V(X).

Section C

Answer any TWO questions:

(2x20=40)

19 (a) For any two subsets of (not disjoint) of a sample space, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(b) The probability that a man survives for another 30 years hence is $\frac{4}{5}$ and that for his wife is $\frac{7}{12}$. Assuming that their survival are independent, find probability that (i) both will survive for next 30 years, (ii) atleast one will survive for the next 30 years and (iii) none will survive for the next 30 years.

20. (a) For any two events A, B and C, show that

(i) $P(A \cup B | C) = P(A | C) + P(B | C) - P(A \cap B | C)$

(ii) $P(A \cap B^c | C) + P(A \cap B | C) = P(A | C)$

(b) Four cards are drawn at random from a pack of 52 cards. Find the probability that the cards drawn are

(1) A king, a queen, a jack and an ace.

(2) Two kings and two queens.

(3) Two black and two red cards.

(4) Two hearts and two diamonds. (10+10)

21 (a) State and prove Bayes Theorem, and mention its importance.

(b) The contents of the three Urns U_1 , U_2 and U_3 are as follows:

U_1 : 1 White, 2 Black and 3 Red marbles

U_2 : 2 White, 1 Black and 1 Red marbles

U_3 : 4 White, 5 Black and 3 Red marbles. One Urn is chosen at random and two marbles are drawn from it. They happen to be white and red. What is the probability that they have come from U_2 ?

22 (a) State and prove Chebyshev's inequality. What is its importance?

(b) If $f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$ is a pdf, find (i) the value of 'k' (ii) mean

and variance. (10+10)